Calculus Warm Up #10-4

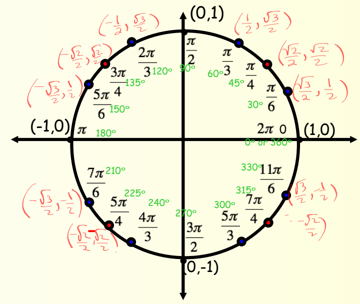
Find:

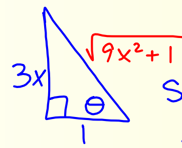
1. arccot $(-\sqrt{3})$

2. arctan 0

3. arccos $\left(\cos \frac{7\pi}{2}\right)$

4. $\sec(\underbrace{\arctan 3x}_{\theta}) \longrightarrow$





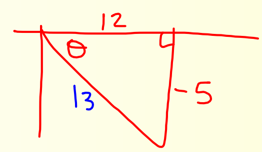
$\sec\theta = \sqrt{9x^2+1}$

$\frac{h}{a}$

5. $\csc(\underbrace{\arctan(-\frac{5}{12})}_{\theta}) \longrightarrow$ tan - in QII & IV, but $-\frac{\pi}{2} < \tan < \frac{\pi}{2}$, so θ in QIV:

$\frac{h}{o}$

$\boxed{\csc\theta = -\frac{13}{5}}$



HW Questions: p 446 #45-65 odd

In Exercises 45–50, use implicit differentiation to find dy/dx and evaluate the derivative at the indicated point.

45. $\sin x + \cos 2y = 2$ $\quad \left(\frac{\pi}{2}, 0\right)$

47. $\tan(x + y) = x$ $\quad (0, 0)$

49. $x \cos y = 1$ $\quad \left(2, \frac{\pi}{3}\right)$

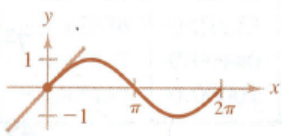
In Exercises 51 and 52, show that the function satisfies the differential equation.

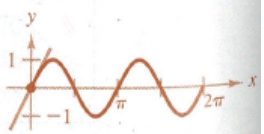
51. $y = 2\sin x + 3\cos x$

$y'' + y = 0$

In Exercises 53 and 54, find the slope of the tangent line to the given sine function at the origin. Compare this value to the number of complete cycles in the interval $[0, 2\pi]$.

53. (a) $y = \sin x$ (b) $y = \sin 2x$





In Exercises 55–62, evaluate each limit, using L'Hôpital's Rule when necessary.

55. $\lim_{x \to 0} \dfrac{\sin 2x}{\sin 3x}$

57. $\lim_{x \to 0} \dfrac{x - \tan x}{x - \sin x}$

59. $\lim_{\theta \to 0} \dfrac{1 - \cos 2\theta}{4\theta^2}$

61. $\lim_{x \to \infty} x \sin \dfrac{1}{x}$

In Exercises 63–66, sketch the graph of each function on the indicated interval, making use of relative extrema and points of inflection.

63. $f(x) = 2\sin x + \sin 2x \qquad [0, 2\pi]$

intercepts

$(0,0)\ (\pi,0)$
$(2\pi,0)$

$0 = 2\sin x + 2\sin x\cos x$
$0 = 2\sin x(1 + \cos x)$
$x = 0, \pi, 2\pi \qquad \cos x = -1,\ x = \pi$

$f'(x) = 2\cos x + 2\cos 2x$
$0 = 2\cos x + 2(2\cos^2 x - 1)$
$2(2\cos^2 x + \cos x - 1)$
$2(2\cos x - 1)(\cos x + 1)$

$x = \frac{\pi}{3}, \frac{5\pi}{3}, \pi$

f': $\frac{\pi}{4}$, $\frac{\pi}{2}$, $\frac{3\pi}{2}$, $\frac{7\pi}{4}$
$0 \oplus \frac{\pi}{3} \ominus \pi \ominus \frac{5\pi}{3} \oplus 2\pi$

Max @ $\left(\frac{\pi}{3}, f\left(\frac{\pi}{3}\right)\right)$ Min @ $\left(\frac{5\pi}{3},\right.$

$f''(x) = -2\sin x - 4\sin 2x$
$0 = -2(\sin x + 2(2\sin x\cos x))$
$0 = -2\sin x(1 + 4\cos x)$
$x = 0, \pi, 2\pi \qquad \cos x = -\frac{1}{4}$
QII
QIII
$x \approx 1.8$ STO→ A
$x \approx 4.5$ STO→ B

f'': 1, 1.8, 2, 3.1, 4, 4.5, 5, 6.3
$0 \ominus A \oplus \pi \quad B \quad 2\pi$

test $f''(1) = -$
$f''(2) =$
$f''(4) =$
$f''(5) =$

PI's @ (A ,
(

In Exercises 63–66, sketch the graph of each function on the indicated interval, making use of relative extrema and points of inflection.

65. $f(x) = x - \sin x \qquad [0, 4\pi]$

Intercepts: $(0,0)$

$f'(x) = 1 - \cos x$
$0 = 1 - \cos x$
$\cos x = 1 \qquad x = 0, 2\pi, 4\pi$

f': π, 3π
$0 \oplus 2\pi \oplus 4\pi$

f increasing on $[0, 4\pi]$,
no extrema

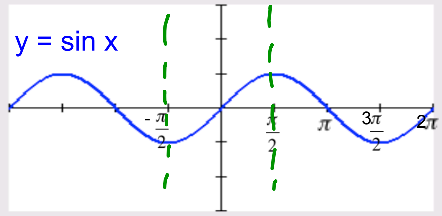
$f''(x) = \sin x$
$0 = \sin x$
$x = 0, \pi, 2\pi, 3\pi, 4\pi$

f'': $\frac{\pi}{2}$, $\frac{3\pi}{2}$, $\frac{5\pi}{2}$, $\frac{7\pi}{2}$
$0 \oplus \pi \ominus 2\pi \oplus 3\pi \ominus 4\pi$

PI's: (π,π); $(2\pi, 2\pi)$; $(3\pi, 3\pi)$

Is y = sin x a function?
Yes

Is its inverse a function?
No, original is not one-to-one.

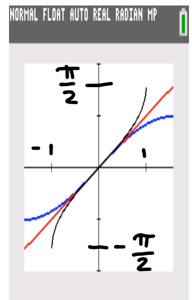


How could we restrict the domain so that it is as large as possible and its inverse is a function? dom: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ range: $[-1, 1]$

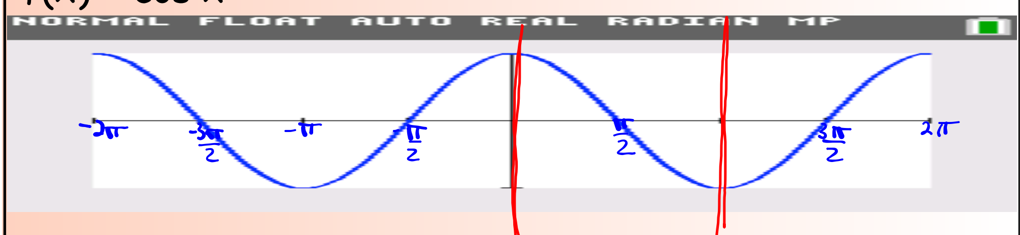
$f^{-1}(x) =$ arc sin x

dom: $[-1, 1]$ range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ → an angle in Q IV or Q I

What would the graph look like?



f(x) = cos x



How would you restrict the domain of f(x) = cos x so that its inverse is also a function? dom: $[0, \pi]$ range: $[-1, 1]$

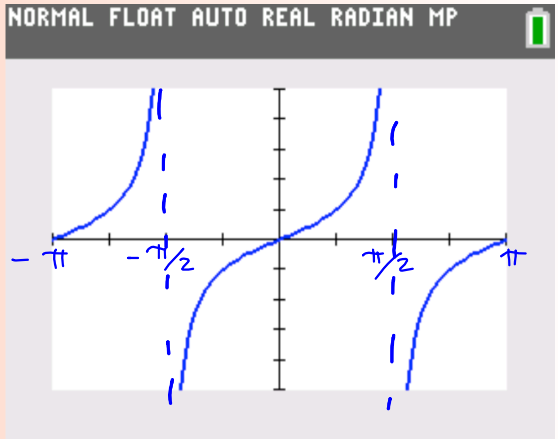
$f^{-1}(x) =$ $\cos^{-1}(x)$

dom: $[-1, 1]$

range: $[0, \pi]$

Angle in Q I & II

$f(x) = \tan x$

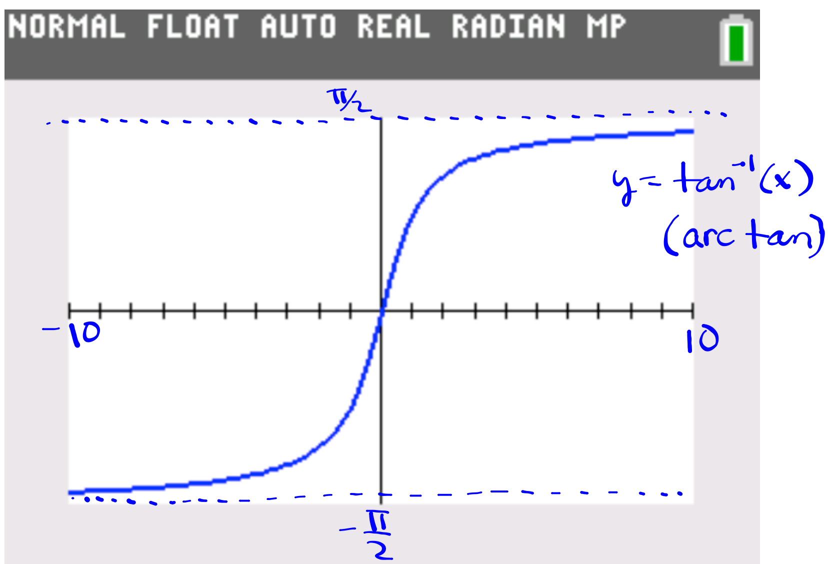


How would you restrict the domain of $f(x) = \tan x$ so that its inverse is also a function? dom: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ range: $\mathbb{R}$

$f^{-1}(x) =$ arctan x

dom: $\mathbb{R}$

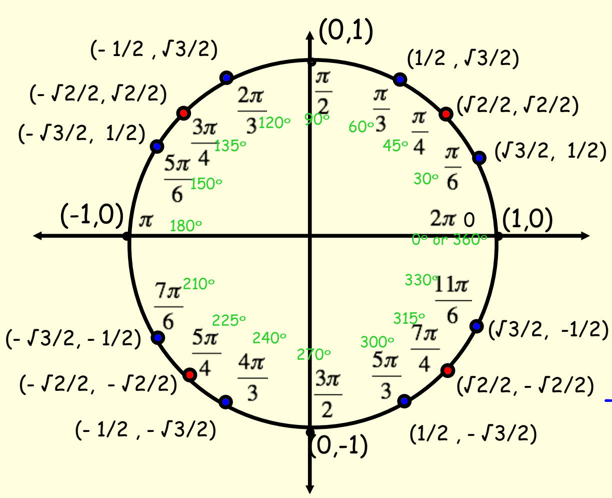
range: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$



Definition of the Inverse Trig Functions

Function	Domain	Range
y = arcsin x if and only if x = sin y	$-1 \le x \le 1$	$\frac{-\pi}{2} \le y \le \frac{\pi}{2}$ θ
y = arccos x if and only if x = cos y	$-1 \le x \le 1$	$0 \le y \le \pi$
y = arctan x if and only if x = tan y	$-\infty < x < \infty$ all reals	$\frac{-\pi}{2} < y < \frac{\pi}{2}$

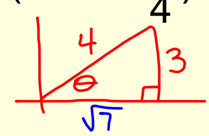
(See summary box on p. 459)



Find:

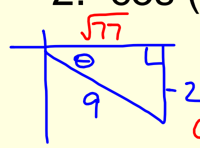
1. $\tan\left(\arcsin \frac{3}{4}\right)$ QI

$\tan\theta = \frac{3\sqrt{7}}{7}$



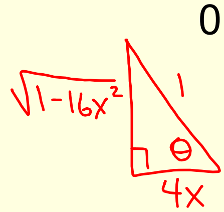
2. $\cos\left(\sin^{-1}\left(-\frac{2}{9}\right)\right)$

θ in Q IV



$\cos\theta = \frac{\sqrt{77}}{9}$

3. $\cot(\arccos 4x)$
$0 \le x \le \frac{1}{4}$



$\cot\theta = \frac{4x}{\sqrt{1-16x^2}}$

8.5 Derivatives of inverse trig functions

Let u be a differentiable function of x. p. 463

$$\frac{d}{dx}[\arcsin u] = \frac{u'}{\sqrt{1-u^2}} \qquad \frac{d}{dx}[\arccos u] = \frac{-u'}{\sqrt{1-u^2}}$$

$$\frac{d}{dx}[\arctan u] = \frac{u'}{1+u^2} \qquad \frac{d}{dx}[\text{arccot } u] = \frac{-u'}{1+u^2}$$

$$\frac{d}{dx}[\text{arcsec } u] = \frac{u'}{|u|\sqrt{u^2-1}} \qquad \frac{d}{dx}[\text{arccsc } u] = \frac{-u'}{|u|\sqrt{u^2-1}}$$

(a) $\frac{d}{dx}[\arctan(3x)] = \frac{3}{1+9x^2}$

$u = 3x$

$u' = 3$

(b) $\frac{d}{dx}[\arcsin \sqrt{x}]$ $u = \sqrt{x}$ $u' = \frac{1}{2\sqrt{x}}$

$= \frac{1}{2\sqrt{x}} \cdot \frac{1}{\sqrt{1-(\sqrt{x})^2}} = \frac{1}{2\sqrt{x-x^2}}$

(c) $\frac{d}{dx}[\text{arcsec } e^{2x}] = \frac{2\cancel{e^{2x}}}{\cancel{e^{2x}}\sqrt{e^{4x}-1}}$

$u = e^{2x}$

$u' = 2e^{2x}$

$= \frac{2}{\sqrt{e^{4x}-1}}$

p. 466

1. $\frac{d}{dx}[cu] = cu'$
2. $\frac{d}{dx}[u \pm v] = u' \pm v'$
3. $\frac{d}{dx}[uv] = uv' + vu'$
4. $\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{vu' - uv'}{v^2}$
5. $\frac{d}{dx}[c] = 0$
6. $\frac{d}{dx}[u^n] = nu^{n-1}u'$
7. $\frac{d}{dx}[x] = 1$
8. $\frac{d}{dx}[|u|] = \frac{u}{|u|}(u'),\ u \neq 0$
9. $\frac{d}{dx}[\ln u] = \frac{u'}{u}$
10. $\frac{d}{dx}[e^u] = e^u u'$
11. $\frac{d}{dx}[\sin u] = (\cos u)u'$
12. $\frac{d}{dx}[\cos u] = -(\sin u)u'$
13. $\frac{d}{dx}[\tan u] = (\sec^2 u)u'$
14. $\frac{d}{dx}[\cot u] = -(\csc^2 u)u'$
15. $\frac{d}{dx}[\sec u] = (\sec u \tan u)u'$
16. $\frac{d}{dx}[\csc u] = -(\csc u \cot u)u'$
17. $\frac{d}{dx}[\arcsin u] = \frac{u'}{\sqrt{1-u^2}}$
18. $\frac{d}{dx}[\arccos u] = \frac{-u'}{\sqrt{1-u^2}}$
19. $\frac{d}{dx}[\arctan u] = \frac{u'}{1+u^2}$
20. $\frac{d}{dx}[\text{arccot } u] = \frac{-u'}{1+u^2}$
21. $\frac{d}{dx}[\text{arcsec } u] = \frac{u'}{|u|\sqrt{u^2-1}}$
22. $\frac{d}{dx}[\text{arccsc } u] = \frac{-u'}{|u|\sqrt{u^2-1}}$

Differentiate $y = \arcsin x + x\sqrt{1 - x^2}$.

$$y' = \frac{1}{\sqrt{1-x^2}} + x \cdot \frac{1}{\cancel{2}}(1-x^2)^{-1/2}(-\cancel{2}x) + \frac{\sqrt{1-x^2}}{1} \cdot \frac{\sqrt{1-x^2}}{\sqrt{1-x^2}}$$

$$\frac{1}{\sqrt{1-x^2}} - \frac{x^2}{\sqrt{1-x^2}} + \frac{1-x^2}{\sqrt{1-x^2}}$$

$$\frac{2-2x^2}{\sqrt{1-x^2}} \cdot \frac{\sqrt{1-x^2}}{\sqrt{1-x^2}}$$

$$\frac{2(1-x^2)\sqrt{1-x^2}}{1-x^2} \longrightarrow \frac{\text{Simplified}}{2\sqrt{1-x^2}}$$

HW: p. 467

39 - 61 odd

and part 1 of the Ch. 8 review worksheet to keep you entertained over the long weekend.